Factoring Summary

Before factoring any polynomial, write the polynomial in **descending order** of one of the variables.

1. Factor out the Greatest Common Factor (GCF). Look for this in **<u>every</u>** problem. This includes factoring out a -1 if it precedes the leading term.

   Example: $-3x^2 + 12x - 18 = -3(x^2 - 4x + 6)$

2. If there are **<u>FOUR TERMS</u>**, try to factor by grouping (GR).

   Example: $x^3 + 6x^2 - 2x - 12$

   $\underline{x^3 + 6x^2}\quad \underline{-2x - 12} =$ *group the first two terms, last two terms*

   $x^2(x+6) - 2(x+6) =$ *factor out GCF from each grouping*

   $(x+6)(x^2 - 2)$ *factor out the common grouping*

3. If there are **<u>TWO TERMS</u>**, look for these patterns:

x	x^2	x^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	
10	100	
11	121	
12	144	
13	169	
14	196	
15	225	

   a. The difference of squares (DOS) factors into conjugate binomials:

   $a^2 - b^2 = (a-b)(a+b)$

   Example: $9x^4 - 64y^2 = (3x^2 - 8y)(3x^2 + 8y)$

   Note: a variable is a perfect square if the exponent is even

   b. The sum of squares does not factor:

   $a^2 + b^2$ is prime

   Example: $9x^4 + 64y^2$ is PRIME

   c. The sum of cubes (SOC) or difference of cubes (DOC) factors by these patterns: each type contains a binomial (small bubble) times a trinomial (large bubble). Only the sign patterns differ between sum of cubes and difference of cubes.

   $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

   Example : $(8x^3 + 27) = (2x+3)(4x^2 - 6x + 9)$

   $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

   Example : $(64x^6 - 125y^3) = (4x^2 - 5y)(16x^4 + 20x^2y + 25y^2)$

   Note: a variable is a perfect cube if the exponent is a multiple of three

4. If there are **<u>THREE TERMS</u>**, look for these patterns:

a. Quadratic trinomials of the form $ax^2 + bx + c \;\; where \;\; a = 1$ ($QT \;\; a = 1$))factor into the product of two binomials (double bubble) where the factors of c must add to b.

Example: $x^2 - 4x - 12 = (x-6)(x+2)$

b. Quadratic trinomials of the form $ax^2 + bx + c \;\; where \;\; a \neq 1$ ($QT \;\; a \neq 1$) eventually factor into the product of two binomials (double bubble), but you must first find the factors of *ac* that add to *b*, rewrite the original replacing *b* with these factors of *ac*, then factor by grouping to finally get to the double bubble.

Example:

$9x^2 + 15x + 4 \qquad ac = (9)(4) = 36$

$factors \;\; of \;\; 36 \;\; that \;\; add \;\; to \;\; 15: \;\; 12 \;\; and \;\; 3$

$9x^2 + 12x + 3x + 4 =$

$3x(3x+4) + 1(3x+4) =$

$(3x+4)(3x+1)$

c. Quadratic square trinomials (*QST)* of the form $ax^2 + bx + c$ may factor into the square of a binomial. Look for the pattern where two of the terms are perfect squares, and the remaining term is twice the product of the square root of the squares:

$a^2 \pm 2ab \pm b^2 = (a \pm b)^2$

Example: $16x^2 - 40x + 25 = (4x-5)^2$

5. Factor all expressions completely. Sometimes, you will need to use two or three types of factoring in a single problem.

Example:

$-2x^4 + 32 = \qquad factor \;\; out \; the \;\; GCF \;\; of \;\; -2$

$-2(x^4 - 16) = \qquad factor \;\; the \;\; difference \;\; of \;\; squares$

$-2(x^2 - 4)(x^2 + 4) = \qquad factor \;\; the \;\; remaining \;\; difference \;\; of \;\; squares$

$-2(x-2)(x+2)(x^2+4) \qquad (remember \;\; that \;\; the \;\; sum \;\; of \;\; squares \;\; is \;\; prime)$