

Methods for Solving Quadratic Equations

Quadratic equations are of the form $ax^2 + bx + c = 0$, where $a \neq 0$

Quadratics may have two, one, or zero real solutions.

1. FACTORING

Set the equation equal to zero. If the quadratic side is factorable, factor, then set each factor equal to zero.

Example: $x^2 = -5x - 6$

Move all terms to one side $x^2 + 5x + 6 = 0$

Factor $(x + 3)(x + 2) = 0$

Set each factor to zero and solve $x + 3 = 0$ $x + 2 = 0$
 $x = -3$ $x = -2$

2. PRINCIPLE OF SQUARE ROOTS

If the quadratic equation involves a **SQUARE** and a **CONSTANT** (no first degree term), position the square on one side and the constant on the other side. Then take the square root of both sides.

(Remember, you cannot take the square root of a negative number, so if this process leads to taking the square root of a negative number, there are no real solutions.)

Example 1: $x^2 - 16 = 0$

Move the constant to the right side $x^2 = 16$

Take the square root of both sides $\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$, which means $x = 4$ and $x = -4$

Example 2: $2(x + 3)^2 - 14 = 0$

Move the constant to the other side $2(x + 3)^2 = 14$

Isolate the square $(x + 3)^2 = 7$ (divide both sides by 2)

Take the square root of both sides $\sqrt{(x + 3)^2} = \pm\sqrt{7}$

$x + 3 = \pm\sqrt{7}$

Solve for x $x = -3 \pm \sqrt{7}$

This represents the exact answer.

Decimal approximations can be found using a calculator.

3. COMPLETING THE SQUARE

If the quadratic equation is of the form $ax^2 + bx + c = 0$, where $a \neq 0$ and the quadratic expression is not factorable, try completing the square.

Example:

$$x^2 + 6x - 11 = 0$$

****Important:** If $a \neq 1$, divide all terms by “a” before proceeding to the next steps.

Move the constant to the right side
and supply a blank on each side

$$x^2 + 6x + \underline{\hspace{1cm}} = 11 + \underline{\hspace{1cm}}$$

Find half of b , which means $\frac{b}{2}$:

$$\frac{6}{2} = 3$$

Square half of b : $\left(\frac{b}{2}\right)^2$:

$$3^2 = 9$$

Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation

$$x^2 + 6x + 9 = 11 + 9$$

Factor the quadratic side

$$(x + 3)(x + 3) = 20$$

(which is a perfect square because you just made it that way!)

Then write in perfect square form

$$(x + 3)^2 = 20$$

Take the square root of both sides

$$\sqrt{(x + 3)^2} = \pm\sqrt{20}$$

$$x + 3 = \pm\sqrt{20}$$

$$x = -3 \pm \sqrt{20} \quad \text{Simplify the radical}$$

$$x = -3 \pm 2\sqrt{5}$$

Solve for x

This represents the exact answer.

Decimal approximations can be found using a calculator: 1.472 and -7.472

4. QUADRATIC FORMULA

Any quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$ can be solved for both real and imaginary solutions using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$$x^2 + 6x - 11 = 0 \quad (a = 1, \quad b = 6, \quad c = -11)$$

Substitute values into the quadratic formula:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-11)}}{2(1)} \rightarrow x = \frac{-6 \pm \sqrt{36 + 44}}{2} \rightarrow x = \frac{-6 \pm \sqrt{80}}{2} \quad \text{simplify the radical}$$

$$x = \frac{-6 \pm 4\sqrt{5}}{2} \rightarrow x = -3 \pm 2\sqrt{5} \quad \text{This is the final simplified EXACT answer}$$